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Application of Direct Methods in Optimal Path Planning of Redundant Cooperative Robots

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Abstract—**This paper introduces a numerical technique for solving the optimal path planning problem of redundant cooperative manipulators. The technique is based on converting Abstract Optimization Problems to Parametric Optimization Problems using Direct Methods. In this approach, the state and control variables are expanded in finite numbers of Legendre Polynomials and the kinematics of cooperative system as well as performance index of optimization problem are approximated. Application of this method results in the transformation of nonlinear differential and integral expressions into nonlinear algebraic expressions. The optimal states are then determined by solving a set of nonlinear equations in Legendre coefficients. The approach is applied to a cooperative manipulator system consisting of two 3-DOF serial manipulators jointly carrying an object. The results are compared with those obtained from solving a Two Point Boundary Value Problem, as an abstract optimization approach.**

Keywords- Direct Methods; Path Planning; Redundant Cooperative Manipulators; Parametric Optimization; Legendre Polynomials.

I. INTRODUCTION

The capabilities of single robot arms are insufficient for some types of complex tasks. Two or more cooperative robot arms are, therefore, needed to extend the range of potential applications and to handle complicated and dexterous tasks skillfully. When two (or more) robots are cooperating, (a) closed kinematic chain(s) will be generated. Regarding the kind of object grasping, the closed chain is usually redundant. Redundant manipulators can achieve additional tasks by utilizing their degrees of redundancy. However, as a result of kinematic redundancy, path planning for redundant manipulators is complicated. Path planning as redundancy resolution deals with selecting an individual configuration in each instant of time among all possible ones. The common idea in redundancy resolution is that redundancy should be solved in such a way that the mechanism optimizes a performance index of the system while carrying out its given duty.

Let $\mathbf{q} \in \Re^n$ be the joint space coordinates of a redundant cooperative robot system. Also, $\mathbf{X} \in \Re^m$ represents the variables of end effectors, which jointly carry the object and is called *end effector space* in this paper. This vector should not be confused with the vector of object task space. In single robot systems, these two are usually the same. It is worth noting that the number n represents the summation of joint variables of all robots involved, and is different from the number of DOF of the system.

Kinematic function $\mathbf{F}: \Re^n \to \Re^m$ describes the relationship of $\mathbf{X}$ and $\mathbf{q}$ variables as:

$$\mathbf{F}(\mathbf{q}) = \mathbf{X}(t) \tag{1}$$

Therefore, the path planning problem can be stated as: *Find a path* $\mathbf{q}(t)$ *in joint space in a manner that it moves all end effectors along the specified path* $\mathbf{X}(t)$ *in the end effector space* $(t \in [t_0, t_f])$.

Equation (1) does not have an easy solution since it involves triangular nonlinear algebraic equations. Additionally, due to redundancy, it is a set of under-determined equations; *i.e.*, the number of unknown variables exceeds the number of equations $(m < n)$. This equation can be stated at velocity level as

$$\mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \dot{\mathbf{X}}(t) \tag{2}$$

in which $\mathbf{J} = \partial \mathbf{F} / \partial \mathbf{q}$ is the Jacobian matrix of cooperative manipulator system.

Numerous studies on redundant manipulators have involved instantaneous redundancy resolution at velocity level using pseudo inverse of Jacobian matrix [1-3]. A lot of effort has been put into finding the null space solution including the approaches of least square joint velocities. All the methods developed can only rely on local information, which results in local optimal solutions. An alternative method involves the optimization of an integral performance index with the forward kinematics as constraints. The integral is over the length of the path. Therefore, the history of cost function is taken into account, which yields a global optimal. This global method is based on calculus of variations and leads to a set of ordinary differential equations (ODEs) with split boundary conditions (BCs). Accordingly, to obtain the optimal solution, one should solve a boundary value problem [4-6].