A Direct Algorithm to Compute Switching Curve for Time Optimal Motion of Cooperative Multi-Manipulators Moving on Specified Path

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Abstract—**For more than two decades it is known that the time optimal problem for a manipulator along specified path is a bang-bang in term of acceleration along the path and the switching points can be found by phase plane analysis. Despite great advancements, no direct method is still available for finding switching points and constructing switching curve. All proposed methods are based on search algorithms, in which, one has to:**

-Search the whole phase plane to establish the boundary of non feasible area in which end effector can not follow the path.

-Find the critical points by numerical calculation of the slope of the non- feasible boundary.

This paper is concerned with developing a direct method to find the critical points and construction of switching curve for cooperative multi manipulator systems. To this end, a rigorous study of the characteristics of the critical points and the switching curve is presented and based on that a direct algorithm is introduced.

I. Introduction

A Cooperative Multi Manipulator System (CMMS) is defined as a system of multiple robots handling a common object (Figure 1). Application of multiple robots in an assembly line working on an object at a similar time, or, handling a big object with several manipulators could be considered as examples of such systems.

Many researchers studied the problem of minimum time tracking of a manipulator on a specified path. Bobrow et al. [1] proposed a method for time-optimal control of serial manipulators along a specified path. Considering that the solution is bang-bang, the method reduces the problem into calculating the maximum and minimum acceleration along the trajectory in each step, and to find the switching points. They used a geometric approach in the phase plane to find the switching points. According to their method, to find switching points, one has to construct the boundary of Non-Feasible Region (NFR) in phase plane, above which no acceleration can hold the end-effector on path. Their algorithm for calculating the boundary of non-feasible area was based on searching $\dot{s}_{\max}$ for each s, sweeping the whole phase plane. On the other hand, they suggested a shooting method to find switching points, in which one has to find a solution trajectory which comes in contact with the non-feasible boundary without crossing it, which numerically is a very difficult and expensive task to do.

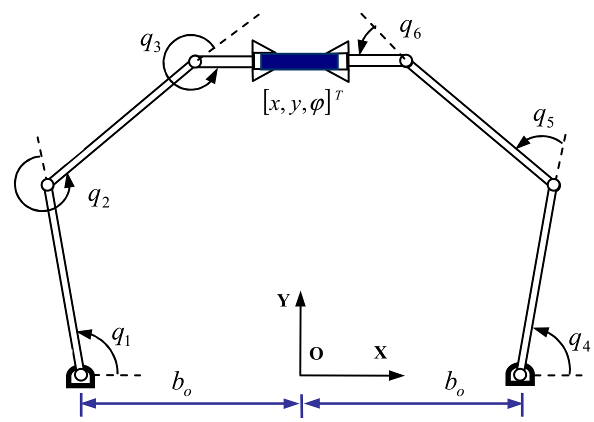

Fig. 1. Schematic diagram of the CMMS

This method was further developed by Pfeiffer and Johanni [2]. Taking advantage of characteristics of the boundary of NFR, they presented a method for direct calculation of that and to find the switching point on the boundary for serial manipulators.

They stated that switching points might occur on the boundary of NFR at critical points, where the slope of non-feasible region boundary minus the value $\frac{\ddot{s}}{\dot{s}}$ changes sign. This advancement considerably reduced the numerical effort for serial manipulators.

Ma and Watanabe [3]-[8] developed this method for serial redundant manipulators with different secondary constraints such as heat characteristics and kinematic characteristics. Galicki [9]-[11] also studied the problem of minimum time motion of redundant manipulators. Park and Bobrow [12] studied the time optimal problem of manipulators with obstacle avoidance. Cho et al. [13] studied the time optimal problem for manipulators imposing acceleration constraint.

The problem of minimum time motion along specified path for cooperative manipulators was also studied by several researchers. McCarthy and Bobrow [14] proved that

Manuscript received March 18, 2007.

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