Wednesday, November 19

11:15am-12:45pm  Balcony A, Mezzanine Level

DSC-15: Robot Actuators
Sponsored by Dynamic Systems and Control - Robotics Panel
Chair: Shane Farritor
Co-chair: Panayiotis Shiakolas, University of Texas at Arlington
IMECE2003-41064 On a Finger Model Actuated with Shape Memory Alloy Artificial Muscles
Veturia V. Chirou, Institute of Solid Mechanics Romanian Academy, Romania; Corneliu Mihai Niculescu, Royal Institute of Technology, Dept. of Production Engineering, Sweden; Ligia I. Munteanu, Traian A. Badea, Institute of Solid Mechanics Romanian Academy, Romania
IMECE2003-41131 Design and Construction of a Novel Tubular Linear Motor with Controller for Robotics Applications
W. Kim, Bryan C. Murphy, Texas A&M University
IMECE2003-41605 Vast DOF Wet Shape Memory Alloy Actuators Using Matrix Manifold and Valve System
Stephen Mascauro, H. Harry Asada, Massachusetts Institute of Technology
Justin W. Raade, Timothy G. McGee, Homayoon Kazerooni, University of California-Berkeley
IMECE2003-42603 Theoretical Analysis and Experimental Verification of a Monopropellant Driven Free Piston Hydraulic Pump
Timothy G. McGee, Justin W. Raade, Homayoon Kazerooni, University of California-Berkeley
IMECE2003-42603 Application of the Extended Kalman Filter to Control a Shape Memory Alloy Arm
Mohammad Elahinia, Virginia Tech; Hashem Ashrafuon, Villanova University, Mehdi Ahmadian, Virginia Polytechnic Institute and State University; Daniel J Irman, Virginia Tech

2:00pm-3:30pm  Balcony A, Mezzanine Level

DSC-16: Robot Design, Modeling and Identification
Sponsored by Dynamic Systems and Control - Robotics Panel
Chair: Mircea Badescu, Rutgers, The State University of New Jersey
Co-chair: Dr. Venkat Krovii
IMECE2003-42963 Workspace Analysis of Reconfigurable Robotic Arms Using Parallel Platforms as Modules
Mircea Badescu, Rutgers, The State University of New Jersey; Constantinos Mavroidis, Rutgers University
IMECE2003-43015 Modeling of Wheeled Mobile Robot on Rough Terrain
Meihua Tai, Polytechnic University
IMECE2003-41246 Dynamic Analysis of a Flexible Manipulator with Passive Constrained Layer Damping
J. Q. Zou, The Hong Kong Polytechnic University, China; Eric HK Fung, H.W.J. Lee, The Hong Kong Polytechnic University, China
IMECE2003-41501 Evolutionary Based Optimal Synthesis of Four-Bar Mechanisms
Panayiotis Shiakolas, University of Texas at Arlington
Panayiotis Shiakolas, University of Texas at Arlington
IMECE2003-42642 The Spatial Motion of a Compliantly Suspended Rigid Body Constrained by Multi-Point Frictional Contact
Shuguang Huang, Prof. Joseph M. Schimmels, Marquette University

3:45pm-5:15pm  Balcony A, Mezzanine Level

DSC-17: Robot Control
Sponsored by Dynamic Systems and Control - Robotics Panel
Chair: Jairo T. Moura, Brooks Automation Inc.
Co-chair: Dr. Martin Hosek, Brooks-PRI Automation, Inc.
IMECE2003-42663 Stability Analysis of a Class of Nonlinear Controllers
Thambirajah Ravichandran, University of Waterloo, Canada; Glenn R. Heppler, Systems Design Engineering, Canada; David W. L. Wang, University of Waterloo, Canada
IMECE2003-42642 Controller Design for a Non-Redundant Cable Robot Under Input Constraint
Soyeck Oh, Sunil K Agrawal, University of Delaware
IMECE2003-42608 A Unified Approach for Independent Manipulator-Joint Acceleration Control and Observation
E.M. Elbenni, University of Windsor, Canada; A.S. Zaki, Cairo University, Egypt; W.H. ElMaraghy, University of Windsor, Canada
IMECE2003-42028 Relay Feedback PID Tuning of a Parallel Manipulator
Zonghua Wang, B.W. Suroger, Queen's University, Canada; George K.I. Mann, Memorial University, Canada
IMECE2003-43103 Modeling of a Flexible Link in Contact: Strain Vs. Force Output
Kamyar Zaree, David W. L. Wang, Glenn R. Heppler, University of Waterloo, Canada
IMECE2003-41249 Robust and Adaptive Resolved Motion Control of a Hydraulic Loader Crane
Henrik C. Pedersen, Henrik C. Pedersen, Aalborg University, Denmark; Brian Nielsen, Torben O. Andersen, Michael R. Hansen, Aalborg University, Denmark

6:45pm-8:15pm  Maryland B, Lobby Level

DSC-18B: Robot Planning
Sponsored by Dynamic Systems and Control - Robotics Panel
Chair: Mircea Badescu, Rutgers, The State University of New Jersey
Co-chair: Venkat Krovii
IMECE2003-41178 Time-Optimum Trajectories for Robots with Multiple End-Effectors
D. Martin Hosek, Brooks-PRI Automation, Inc.
IMECE2003-42100 Optimal Path Planning of Redundant Cooperative Robots Under Equality and Inequality Constraints
Ali Hosseini, Prof. Mehdi Keshmiri, Isfahan University of Technology, Iran
IMECE2003-43493 Dexterous Trajectory Tracking Control of a Mobile Robot
Zhen Zhang, Meng Ji, Nilanjun Sarkar, Vanderbilt University
IMECE2003-42700 Nonholonomic Motion Planning Using Diffusion of Workspace Density Functions
Yu Zhou, Prof. Gregory S. Chirikjian, The Johns Hopkins University
IMECE2003-42710 Modeling Gait Transitions Between Two Periodic Gaits for Quadrupeds and Hexapods
Jian-Nan Lin, Motorola Inc.; Prof. Shin-Min Song, Northern Illinois University
IMECE2003-42868 Recursive Kinematics and Inverse Dynamics for Parallel Manipulators
Waseem A. Khan, McGill University, Canada; Venkat Krovii, SUNY Buffalo; Subir K. Saha, IIT Delhi, India; Jorge Angeles, McGill University, Canada
IMECE2003-42677 GPC-Based Stable Reconfigurable Control
Jianjun Shi, Atul Kelkar, Iowa State University; Don Soloway, NASA Ames Research Center
IMECE2003-42666 Modeling and Control of a Morphing Airfoil
Christopher E. Whitmer, P.T. Vu, Atul Kelkar, Frank Chavez, Iowa State University
IMECE2003-42864 Segmented Wing Aircraft Lateral Directional Flight Control Design with Minimum Drag Constraints
Frank Chavez, Iowa State University
IMECE2003-43594 LOG-Based Robustifying Flight Control System
Dennis Griffin, Iowa State University; Atul Kelkar, Iowa State University
OPTIMAL PATH PLANNING OF REDUNDANT COOPERATIVE ROBOTS UNDER EQUALITY AND INEQUALITY CONSTRAINTS

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ABSTRACT
Using kinematic resolution, the optimal path planning for two redundant cooperative manipulators carrying a solid object on a desired trajectory is studied. The optimization problem is first solved with no constraint. Consequently, the nonlinear inequality constraints, which model obstacles, are added to the problem. The formulation has been derived using Pontryagin Minimum Principle and results in a Two Point Boundary Value Problem (TPBVP). The problem is solved for a cooperative manipulator system consisting of two 3-DOF serial robots jointly carrying an object and the results are compared with those obtained from a search algorithm. Defining the obstacles in workspace as functions of joint space coordinates, the inequality constrained optimization problem is solved for the cooperative manipulators.

Keywords: Redundant Cooperative Robots, Path Planning, Two Point Boundary Value Problem, Equality and Inequalities

1. INTRODUCTION
Cooperation among robots to perform a common task has increasingly developed and created a new field of study in Robotics. The capability of cooperative robot systems to perform complicated, accurate and high performance functions, not expected of single robots, has attracted a lot of attention by researchers. When two (or more) robots are cooperating, they create a closed kinematic chain. Regardless of the kind of object grasping, the closed chain is usually redundant.

In addition to a basic motion task, redundant manipulators can achieve additional tasks by utilizing their degrees of redundancy. However, as a result of kinematic redundancy, path planning for redundant manipulators is a complicated job. There are two main aspects of path planning: motion planning, which deals with the existence of a feasible path; and redundancy resolution, which deals with selecting a single configuration among all possible ones. The common idea in redundancy resolution is that redundancy should be resolved in such a way that the mechanism optimizes a performance measure of system while carrying out its given task.

Two possible approaches to resolving the redundancy are the local optimization methods and the global optimization methods. Numerous studies on redundant manipulators have involved instantaneous redundancy resolution at velocity level using pseudo inverse of Jacobian matrix [1-3]. It has been shown that pure pseudo inverse control to most manipulator geometries is not conservative in that a closed trajectory in the Cartesian space may not necessarily result in a closed joint space trajectory [4]. A Jacobian pseudo inverse approach, modified to include the null space solution, could make the control of a kinematically redundant robot conservative [4,5]. A lot of effort has been put into finding the null space solution including the approaches of least square joint velocities. All the methods developed can only rely on local information. They are not capable of sacrificing local interests so that the trajectories are globally optimized. However, they are adequate for real time implementation. An alternative method involves the optimization of an integral performance index with the forward kinematics as constraints. The integral is over the length of the path. Therefore, the history of cost function is taken into account, which yields a global optimal. The chosen cost function can be a kinematic or kinetic index. The former is a function of kinematic parameters and the kinematic equations are, therefore, considered as constraints of optimization problem; the latter is a function of kinematic parameters such as joint torques or consumed energy and power; hence, the motion equations are used as constraints.

This global method is based on calculus of variation and leads to a set of ODEs with split boundary conditions (BCs). Accordingly, to obtain the optimal solution, one should solve a boundary value problem [6-7]. The price one pays in order to
use a global optimization scheme is the increase in computation time. So, the global methods can not be used in real time applications. Other authors attempted to find the optimal path by solving parameter optimization problem extracted from abstract optimization problem. In these approaches, rather than solving the differential equations, the optimal path is approximated by polynomial of finite dimensions and numerical schemes are used to solve for unknown coefficients [8-9].

A few researchers have dealt with redundancy resolution methods incorporating inequality constraints (ICs) including improved configuration control scheme using active ICs [10], using Kuhn-Tucker condition considering only one active IC at a time [11] and more than one active ICs simultaneously [12].

Section 2 of the paper is dedicated to explaining the problem formulation. The kinematic index of joint velocity norm is used in this study and kinematic redundancy of the closed chain of cooperative robots is solved based on Optimal Control Theory. The theory has been expressed completely by [13] and is extensively used in the present study. In addition, Kuhn-Tucker condition is merged with variational methods to find a global constrained optimal solution to under-determined kinematic equations with nonlinear configuration-dependent ICs. This method is applied when more than one active ICs are imposed simultaneously. In section 3, illustrative examples are described for each case to demonstrate the effectiveness of these approaches. Finally, section 4 provides some conclusions.

2. OPTIMAL PATH PLANNING PROBLEM FORMULATION

2.1. Optimization Under Equality Constraints

Let \( q \in \mathbb{R}^n \) be the joint space coordinates of a redundant cooperative robot system. Also, \( X \in \mathbb{R}^m \) represents the variables of end effectors, which jointly carry the object and is called end effector space in this paper. This vector should not be confused with the vector of object task space. In single robot systems, these two are usually the same. But when the number of robots exceeds unity, each manipulator has its own work space that can be different from the work space of the object. It is worth noting that the number \( n \) represents the summation of joint variables of all robots involved. And, it is different from the number of DOF of the system.

The relation between \( X \) and \( q \) variables is given by the kinematic function \( F : \mathbb{R}^n \rightarrow \mathbb{R}^m \) as:

\[
F(q) = X(t)
\]  

(1)

The path planning problem can be stated as: Find a path \( q(t) \) in joint space in a manner that it moves all end effectors along the specified path \( X(t) \) in the end effector space \((t \in [t_i, t_f])\).

Eq. (1) does not have an easy solution since it involves triangular nonlinear algebraic equations. Additionally, due to redundancy, it is a set of under-determined equations; i.e., the number of unknown variables exceeds the number of equations \((m < n)\). This equation can be stated at velocity level as

\[
J(q)\dot{q} = X(t)
\]  

(2)

in which \( J = \frac{\partial F}{\partial q} \) is the Jacobian matrix of cooperative manipulator system. The general solution to Eq. (2) can be written as the sum of a particular solution and a homogeneous solution:

\[
\dot{q}(t) = \dot{q}_p(t) + \dot{q}_h(t)
\]  

(3.a)

\[
\dot{q}_p(t) = J^+X(t)
\]  

(3.b)

\[
\dot{q}_h(t) = (I_n - J^+J)\beta
\]  

(3.c)

where \( J^+ \) is the pseudo inverse matrix, \( \beta \in \mathbb{R}^n \) is an arbitrary vector and \( I_n \) indicates identity matrix of order \( n \). If the rows of \( J \) are linearly independent, then

\[
J^+ = J^T(JJ^T)^{-1}
\]  

(4)

If \( X \) belongs to the range space of \( J \), then (3) is an exact solution to (2). Else, it is a Least Square Solution. Since Eq. (2) represents a physical model, \( X \) will be defined in the end effector space and (3) is the exact and general solution to kinematic equations. Eq. (3.b) indicates a particular solution, which comes from orthogonal compliment of null space of \( J \). Eq. (3.c) indicates a homogeneous solution, which comes from null space of \( J \) and generates those configurations that make no motion in the end effector space by mapping any arbitrary vector into null space. In fact, \( \dot{q}_h \) indicates the redundancy property of the system and can generate an infinite variety of solutions of \( \dot{q}(t) \). Using this property, one can choose a solution that optimizes an index of system among all possible ones.

In the present study, the following index is chosen

\[
P = \int_{t_i}^{t_f} \dot{q}^T \dot{q} \, dt
\]  

(5)

The resolution in Eq. (3) allows us to transform the optimal path planning problem to an optimal control problem by defining the arbitrary vector from null space as a forcing control vector.

Pontryagin Minimum Principle. A control law \( u^* \in \mathbb{R}^n \), which causes the n-th order system

\[
\dot{x}(t) = f(x, u)
\]  

(6)

to follow an admissible trajectory that minimizes the performance index

\[
P = \int_{t_i}^{t_f} f_o(x, u) \, dt
\]  

(7)

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is sought. Considering adjoint vector of $\psi \in \mathbb{R}^n$, the Hamiltonian of the system is defined as

$$\mathcal{H}(x, u, \psi) = f_0 + \psi^T f$$  \hspace{1cm} (8)$$

The necessary conditions for $u^*$ to be an optimal control involve:

$$\dot{x}^*(t) = \frac{\partial \mathcal{H}}{\partial x}(x^*, u^*, \psi^*)$$  \hspace{1cm} (9.a)$$

$$\dot{\psi}^*(t) = -\frac{\partial \mathcal{H}}{\partial u}(x^*, u^*, \psi^*)$$  \hspace{1cm} (9.b)$$

$$0 = \frac{\partial \mathcal{H}}{\partial \psi}(x^*, u^*, \psi^*)$$  \hspace{1cm} (9.c)$$

$$\mathcal{H}(x^*, u^*, \psi^*) \leq \mathcal{H}(x, u, \psi)$$  \hspace{1cm} (9.d)$$

for all $t \in [t_0, t_f]$ and

$$\psi^T \dot{\psi} = 0 \hspace{0.5cm} @ \hspace{0.2cm} t = t_0, t_f$$  \hspace{1cm} (10)$$

Eqs. (9.a) through (9.c) are algebraic and differential equations. Solving Eq. (9.c) for $u^*(t)$ and substituting in (9.a) and (9.b), one can calculate optimal path of $x^*(t)$. It should be emphasized that Eqs. (9) and (10) constitute a set of necessary conditions for optimality; these conditions are not, in general, sufficient.

When the Hamiltonian is a linear or quadratic function of $u$, then the nonlinear Eq. (9.c) has an analytic solution to $u^*(t)$. Otherwise, it should be solved numerically for every step of integration of Eqs. (9.a) and (9.b). In our case, the index $P$ is a quadratic function of $q$ and so of $\beta$. Therefore, $\beta$ can be found analytically in a closed form. Rewriting the necessary conditions (9) for the system (3) and solving equation (9.c) for $\beta$, we have:

$$\beta = -\frac{1}{2}[I_n - J^T J] \psi$$  \hspace{1cm} (11)$$

Derivation of Eq. (11) is provided in Appendix.

Eventually, the governing equations of optimal control problem are constituted as follows:

$$\dot{q} = f$$  \hspace{1cm} (12.a)$$

$$\psi = \left[ \frac{\partial f}{\partial q} \right]^T (2f + \psi)$$  \hspace{1cm} (12.b)$$

where

$$f = J^T \dot{x}(t) - \frac{1}{2} \left[ I - J^T J \right] \psi$$  \hspace{1cm} (13)$$

**Boundary Conditions.** Since Eqs. (12.a) and (12.b) have $2n$ entries, the BCs of the same numbers are necessary. The self-evident boundary condition is $F(q_n) = X_n$, which has $m$ entries. Regarding kinematic redundancy, $q_f$ is free at $t = t_f$ and the problem is called a Fixed Time and Free End State Problem [14]. So, $\psi(t_f) = 0$ states $n$ BCs derived from Eq. (10). The remaining BCs are to be obtained from the *transversality condition* at $t = t_0$, which says that $\psi(t_0)$ should orthogonally intersect with the manifold $F(q_n) = X_n$. The normal vectors of the manifold consist of the column vector of $J^T$. So, $\psi(t_0)$ should belong to row space of Jacobian matrix. Using the concepts of Linear Algebra, the transversality condition can be stated mathematically as in [13]

$$[I_n - J^T J] \psi = 0 \hspace{0.5cm} t = t_0$$  \hspace{1cm} (14)$$

It can be shown that when $J$ is full rank, the rank of the coefficient matrix in Eq. (14) is $n - m$. Hence, Eq. (14) has $n - m$ independent BCs. Accordingly, $2n$ BCs have been achieved and the problem has been reduced to a Two Point Boundary Value Problem (TPBVP) with $n$ condition at each end points. A systematic method to TPBVP is the Shooting Method. This technique is used to simulate numerical examples in section 3.

**Numerical Consideration.** The Shooting Method is based on the following general procedure: an initial guess is made for all of unspecified conditions at $t = t_n$. Then, the differential equations are integrated with the specified and guessed BCs. The result at $t = t_i$ is compared with the BCs specified at $t = t_f$. If they meet, then the computation result is the solution; otherwise, the result at $t = t_j$ is used to modify the guess at $t = t_0$ based on the difference at $t = t_f$, and the process will be repeated until the result and the unspecified BCs at $t = t_f$ coincide with each other.

If one solves this TPBVP based on the above-mentioned procedure, then the estimation and modification of $n$ conditions must be repeated, which has a high computation cost. But the computational complexity can be reduced in the case under consideration. If $q_s$ satisfies Eq. (1), then $q_f$ necessarily satisfies the same equations as long as $q(t)$ is governed by Eq. (9). The reverse is also true. Therefore, the BCs can be restated as $n - m$ conditions at $t = t_0$ and $n + m$ conditions at $t = t_f$.

Actually, the boundary conditions can be summarized as follows:

$$\left\{ \begin{array}{l} F(q_f) = X_{n} \\ \psi(t_f) = 0 \\ e = [I_n - J^T (q_n) J (q_n)] \psi(t_0) = 0 \end{array} \right.$$  \hspace{1cm} (15)$$

In fact, the shooting method is a zero finding problem, and we should find the roots of $e$, where $e$ is a function of guessed BCs.
Let \( s \in \mathbb{R}^{n-m} \) be the vector of guessed boundary conditions. Using Taylor series expansion, \( s \) can be modified (for \( e \neq 0 \)) in each iteration by the following recursive formula:

\[
s_{n+1} = s_n - \left( \frac{\partial e}{\partial q} \right)_{e=0} \cdot e_n
\]

(16)

until it meets the desired accuracy. Instead of \( e \), \( \|e\| \) is used as termination condition.

Applying Eq. (16) to find the zeros of \( e(s) = 0 \) may lead to results that are out of the feasible range of guessed variations. In this situation, finding the zeros can be started again with a new guess inside the feasible range. The closer the guessed variables to their extremum values, the faster the convergence.

### 2.2. Optimization Under Inequality Constraints

Eq. (1) describes a basic motion task for a redundant cooperative robot system to follow. As mentioned in section 1, redundancy resolution can be achieved by satisfying additional requirements such as optimizing a proper performance index. Joint limits and obstacle avoidance can be considered also as requirements such as optimizing a proper performance index.

The redundancy resolution problem now can be stated as partial derivatives with respect to joint variables.

\[
r_j(q) \leq 0, \quad i = 1, \ldots, p
\]

(17)

where \( r_j(q) = 0 \) corresponds to the boundary of the obstacle, and \( r_j(q) < 0 \) to the permissible region outside of that boundary.

Therefore, the problem is changed to find a unique set of joint angles that minimizes a proper performance index under equality constraints; i.e., kinematic equation, and ICs of Eqs. (17). The redundancy resolution problem now can be stated as the following constrained optimization problem.

Minimize \( P(q, \dot{q}, \lambda, \mu, t) = \int_{t_i}^{t_f} f_0(q, \dot{q}, t) \, dt \)

Subject to:

\[
\begin{align*}
\dot{g}(q) &= F(q) - X = 0 \\
\dot{r}(q) &\leq 0
\end{align*}
\]

where \( r(q) \in \mathbb{R}^n \) is assumed to have continuous first order partial derivatives with respect to joint variables.

A Lagrangian function is defined as

\[
\mathcal{L}(q, \dot{q}, \lambda, \mu, t) = f_0(q, \dot{q}, t) + \lambda^T g + \mu^T r
\]

(18)

where \( \lambda \in \mathbb{R}^m \) and \( \mu \in \mathbb{R}^l \).

Considering the variation of augmented index

\[
P_a = \int_{t_i}^{t_f} \mathcal{L}(q, \dot{q}, \lambda, \mu, t) \, dt
\]

and applying the Kuhn Tucker condition, the necessary conditions for optimization are:

\[
\frac{\partial P_a}{\partial \dot{q}} - \frac{d}{dt} \left( \frac{\partial P_a}{\partial \dot{q}} \right) + \lambda^T \frac{\partial g}{\partial q} + \mu^T \frac{\partial r}{\partial q} = 0
\]

(19)

\[
g(q) = F(q) - X = 0
\]

(20)

\[
\mu^T r(q) = 0, \quad r(q) \leq 0
\]

(21)

While a manipulator moves around to perform its task, it may encounter the boundary of an obstacle. This makes the corresponding IC active. So, ICs can be partitioned into two categories: Those that are to be taken care of, and those that may be essentially ignored [15]. According to Eq. (21) if \( r_j \) is not active, then \( \mu_j = 0 \). This leads Eq. (17) to the form:

\[
\begin{align*}
[r_j = 0, & \quad \mu_j \neq 0 \\
r_j < 0, & \quad \mu_j = 0
\end{align*}
\]

(22)

This means that all active ICs can be treated as equality constraints (ECs) and are used to find the optimal path.

Defining \( r_s \in \mathbb{R}^l \) as the vector of active ICs, another form of Eq. (19) is obtained as

\[
\frac{\partial P_a}{\partial q} - \frac{d}{dt} \left( \frac{\partial P_a}{\partial \dot{q}} \right) + A^T \left[ \lambda_{(n+l)} \right] = 0
\]

(23)

where

\[
A_{(n+l)\times n} = \begin{bmatrix} \frac{\partial g}{\partial q} \\ \frac{\partial r_s}{\partial q} \\ \frac{\partial r_s}{\partial q} \end{bmatrix}
\]

(24)

and \( \lambda_s \in \mathbb{R}^l \) corresponds to \( r_s \).

Lagrange's multipliers are eliminated form Eq. (23) using \( (A^T)^C \) matrix that satisfies the relation

\[
(A^T)^C A^T = 0
\]

(25)

Premultiplying Eq. (23) by \((A^T)^C\) yields a set of reduced order differential equations as follows:

\[
(A^T)^C \left[ \frac{d}{dt} \left( \frac{\partial P_a}{\partial \dot{q}} \right) - \frac{\partial P_a}{\partial \dot{q}} \right] = 0
\]

(26)

Eq. (26) consists of \( n - m - l \) equations. Adding \( l \) equality constraints and \( l \) active ICs, it can be used to calculate \( n \) unknown \( q_i \) through \( q_s \).

The number of active ICs, \( l \), is not always constant and an intelligent algorithm is used to determine the reduced order equations at each step of integration. At each moment, the number of active constraints should be less than or equal to the number of degree of redundancy (DOR) of the system:

\[
0 \leq l \leq \text{DOR}
\]

(27)

Otherwise, no solution is guaranteed.
3. NUMERICAL RESULTS

In this section, the numerical results are presented for optimal path planning, first under equality constraints and, then, under ICs. A cooperative robot system is employed to generate the optimal joint space variables, which consists of two 3-DOF serial robots carrying a common object (Fig. 1) with the following characteristics:

\[ l_1 = 0.5 \text{ (m)} \] , \[ l_2 = 0.4 \text{ (m)} \] , \[ l_3 = 0.3 \text{ (m)} \]
\[ l_4 = 0.5 \text{ (m)} \] , \[ l_5 = 0.4 \text{ (m)} \] , \[ l_6 = 0.3 \text{ (m)} \]

The distance between the bases of two robots = 1 (m).

The trajectory of the center of mass of the object is known in cartesian space as \([x(t), y(t), \theta(t)]^T\) with respect to the reference coordinate frame attached to the base of Robot 1 (Fig. 1). \( \theta \) is defined as the angle between the line, which connects two end effectors and the horizontal axis of the reference coordinate frame. The object is connected to each end effector by a revolute joint where \( x_1, y_1 \) and \( x_2, y_2 \) are the coordinates of the end effectors P1 and P2 in Fig. (1), respectively. The joint space variables are defined as \( q(t) = [q_1(t), \ldots, q_n(t)]^T \). Having trajectory of the center of mass of the object, the task space of each serial robot; i.e., \( x_1, y_1 \) and \( x_2, y_2 \) can be specified. Hence, \( n = 6 \) and \( m = 4 \).

And so, it has 2 degrees of redundancy.

3.1. Path Planning Under Equality Constraints

The mass center of the object is supposed to move along the following trajectory:

\[
\begin{align*}
    x(t) &= 0.5 \\
    y(t) &= \frac{1}{4}[1 - \cos(\pi t)] , \quad t \in [0,1] \\
    \theta(t) &= 0
\end{align*}
\] (28)

which is a symmetric trajectory with respect to Robots 1 and 2. Since DOR = 2, only two variables are needed to be guessed at \( t = t_f \) to complete BCs at \( t = t_f \). Choosing \( q_{ef} = q_{1f}(t_f) \) and \( q_{ef} = q_{4f}(t_f) \) as components of \( q \), the optimal control problem is solved. As a result of geometric limits of links, \( q_{el} \) and \( q_{el} \) can vary on bounded intervals. These intervals for given lengths are:

\[ q_{el} \in [0.406, 2.198] \quad \text{and} \quad q_{el} \in [0.943, 3.547] \]

So the guessed values must be chosen from the above intervals.

Other variable can be obtained from kinematic equations. It should be mentioned that kinematic equations yield to two sets of elbow up and elbow down solutions for each manipulator based on admissible \( q_{el} \)’s and \( q_{el} \)’s.

The simulation results are presented in Table (1). Also, for the first example, the generated path in joint space and the corresponding overall behavior of manipulators are shown in Figs. (2) and (3), respectively. It is observed that the optimal solutions for Robot 2 are completely symmetric, as expected, with respect to Robot 1’s and obey Eq. (29)

\[
\begin{align*}
    q_4(t) &= \pi - q_1(t) \\
    q_3(t) &= -q_2(t) \\
    q_6(t) &= -q_3(t)
\end{align*}
\] (29)

The obtained results are checked with a search algorithm, which is explained following.

Considering each manipulator as a single robot system, the optimal control problem can be divided into two optimal control problems, one for each robot; while each end effector moves along the same trajectory during cooperation action.

Since each of the serial manipulators has one degree of redundancy, the cost function for each robot can be calculated as a function of one of the corresponding joint space coordinates; e.g. \( q_{el} \) for Robot 1 and \( q_{el} \) for Robot 2.

Cost functions \( P_u \) and \( P_d \) are evaluated for all feasible elbow up and elbow down configurations, correspondingly. Fig. (4) presents \( P_u \) and \( P_d \) versus \( q_{el} \) for Robot 1 and, \( P_u \) and \( P_d \) versus \( q_{el} \) for Robot 2. Since velocities are excessively large in these neighborhoods, the value of \( P \) is also too large in these neighborhoods and the plots of Fig. (4) are zoomed on the lower parts of the curves. It has been observed that the generated optimum paths in the joint space of each manipulator are exactly the same as the joint space optimum paths for cooperating robots.

An interesting point is that if the cost function of cooperative system is separable for each individual robot; i.e.,

\[ f_t = f_0 + f_0 + \ldots \] (30)

then for simplicity, each robot can be treated separately and the simulation results may be used for the whole system. It yields, consequently, a slighter burden of calculation and so, a faster convergence. Table (2) presents the results obtained form this approach. As can be seen, the evaluated optimum indices of each manipulator are exactly the same and equal to one half of its value during cooperation.

Fig. (5) shows an optimal solution for a sophisticated desired trajectory as follows

\[
\begin{align*}
    x(t) &= 0.4 + \frac{1}{5}[1 - \cos(\pi t)] \\
    y(t) &= \frac{1}{5}[1 - \cos(\pi t)] , \quad t \in [0,1] \\
    \theta(t) &= -\frac{\pi}{4}[1 - \cos(\pi t)]
\end{align*}
\] (31)

optimal values for Fig. (5) are:

\[ q_{el} = -0.107 \text{ (rad.)} \]
\[ q_{el} = -2.491 \text{ (rad.)} \]
\[ P = 4.2493 \text{ (rad^2/sec.)} \]
3.2. Path Planning Under ICs

The inequality constrained optimization problem is applied to the above-mentioned 5-DOF system of cooperating robots. The main task is to move the object along the same path of Eq. (28). Let an additional task minimize the cost function in Eq. (5) and the other additional task avoid joint limits or obstacles in the work space, both of which can be expressed as ICs. The illustrated method has been implemented for each of these two kinds of ICs. To shorten the discourse, only the case of obstacle avoidance is mentioned here.

Two obstacles are defined in the work space of the cooperative robot system: 1) A circle with a radius of 0.15 and a center of [0.25, 0.4], which must be avoided by the right hand manipulator and, 2) An ellipse with a center of [0.75, 0.4] and a major axis parallel to y axis, which must be avoided by the left hand manipulator. The lengths of major and minor axes are 0.5 and 0.3.

It is supposed that the third joint of each manipulator must avoid the above-mentioned obstacles. So, ICs are stated as:

\[ r_1 = 1 - \frac{1}{0.15^2} (I_1c_1 + I_2c_{12} - 0.75)^2 - \frac{1}{0.25^2} (I_1s_1 + I_2s_{12} - 0.4)^2 \leq 0 \]

\[ r_2 = 0.15^2 - (I_4c_4 + I_5c_{45} - 0.25)^2 - (I_4s_4 + I_5s_{45} - 0.4)^2 \leq 0 \]  

(32)

where \( s_i = \sin(q_i) \) and \( s_{ij} = \sin(q_i + q_j) \) and so on. Employing the optimal results of the first example of TPBVP as initial conditions, the simulation results are presented in Figs. (6) and (7). The overall behavior of the manipulators in Fig. (6) shows that the object tracks its desired trajectory while robots avoid assumed obstacles. Fig. (8) demonstrates that Eqs. (32) activate at 0.46 and 0.61 sec., correspondingly and, remain active during the rest of the trajectory.

4. CONCLUSIONS

This paper proposed a method of kinematic resolution of redundant cooperative manipulators using Optimal Control Theory with kinematic equations as equality constraints and a cost function of kinematic parameters. The resulting TPBVP's are solved using shooting method. Since the proposed method uses an integral form of cost function, the results are globally optimal. Also, additional tasks such as joint limits and obstacle avoidance are applied as ICs to resolve kinematic redundancy. To this end, an intelligent algorithm is employed to obtain reduced order differential equations utilizing active ICs. The results and some numerical considerations have been presented through numerical simulations for a system containing two 3-DOF cooperating manipulators.

5. REFERENCES


Table 1: Optimization Performance for Cooperative Manipulator
Obtained From TPBVP (\|e\| < 0.01)

<table>
<thead>
<tr>
<th>First guess ((q_{1f} = 0.4, q_{4f} = 2.742))</th>
<th>Number of Iterations</th>
<th>Optimal (q_{1f}) (rad.)</th>
<th>Optimal (q_{4f}) (rad.)</th>
<th>Optimal (P) (rad^2/sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14</td>
<td>0.048</td>
<td>3.094</td>
<td>2.5852</td>
</tr>
<tr>
<td>Second guess ((q_{1f} = 1, q_{4f} = 2.142))</td>
<td>11</td>
<td>0.571</td>
<td>2.57</td>
<td>3.4171</td>
</tr>
</tbody>
</table>

Table 2: Optimization Performance for a Single Robot Obtained From TPBVP (\|e\| < 0.01)

<table>
<thead>
<tr>
<th>First guess ((q_{1f} = 0.4))</th>
<th>Number of Iterations</th>
<th>Optimal (q_{1f}) (rad.)</th>
<th>Optimal (q_{10}) (rad.)</th>
<th>Optimal (q_{20}) (rad.)</th>
<th>Optimal (q_{30}) (rad.)</th>
<th>Optimal (P) (rad^2/sec.)</th>
<th>(P) in Fig. 4 (rad^2/sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>0.048</td>
<td>-0.662</td>
<td>1.377</td>
<td>2.275</td>
<td>1.2924</td>
<td>1.2928</td>
</tr>
<tr>
<td>Second guess ((q_{1f} = 1))</td>
<td>8</td>
<td>0.572</td>
<td>-0.114</td>
<td>2.320</td>
<td>-3.289</td>
<td>1.7085</td>
<td>1.7086</td>
</tr>
</tbody>
</table>

Figure 1: 5-DOF cooperative robot system

Figure 2: Time History of Optimal Joint Angles for Maneuver Defined by Eq. (28)
Figure 3: Optimal Configuration of Cooperative Manipulators during Maneuver (28)

Figure 4: Variation of $P_u$ and $P_d$ of the Robots 1 & 2 w.r.t. $q_{1f}$ & $q_{4f}$
Figure 5: Optimal Configuration of Cooperative Manipulators During Maneuver (31)

Figure 6: Optimal Configuration of Cooperative Manipulators During Maneuver (28) in The Presence of Obstacles

Figure 7: Comparison of Time History of Optimal Joint Angles during Maneuver (28) With and Without Obstacle
APPENDIX

Derivation of Equation (11)

Since Eq. (3.a) is a linear function of $\beta$, we can write

\[
q = f = c + B\beta
\]  

(A.1)

where

\[
B = (I_n - J^TJ)
\]

(A.2)

for $f_0 = q^Tq$ and (A.1), we will have from Eq. (9.c)

\[
\frac{\partial \mathcal{H}}{\partial \beta} = 2 \left( \frac{\partial q}{\partial \beta} \right)^T \dot{q} + \left( \frac{\partial f}{\partial \beta} \right)^T \psi
\]

(A.3)

\[
= 2B^T(c + B\beta) + B^T\psi = 0
\]

Solving (A.3) for $\beta$:

\[
\beta = -(B^TB)^+B^T[c + \frac{1}{2}\psi]
\]

(A.4)

Considering the definition of $B$ and pseudo inverse of a matrix, the following properties can be experienced

\[
i) B^2 = B
\]

\[
ii) B^T = B
\]

\[
iii) B^T = B
\]

\[
av) Bc = 0
\]

(A.5)

Substituting (A.5) in (A.4), the optimal control law will be simplified as:

\[
\beta = -\frac{1}{2}(I - J^TJ) \psi
\]

(A.6)

The above equation shows a necessary condition for optimal control law.