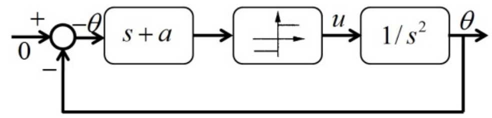
Advanced Dynamics Assignment #3

Due date: Tuesday 95/10/07

Problem 1: the system block diagram shown in the figure represents a block diagram of a satellite control system with a rate feedback and on-off actuator. Derive equations of motion for this system. Determine the equilibrium point of the system. Draw the phase portrait of the system and determine the system's stability.



Problem 2: Consider the nonlinear system

$$\dot{x} = x\left[-1+(a+1)+2y-a(x^2+xy+y^2)\right]$$
$$\dot{y} = y\left[1-a+2(a-1)x+(2a-1)y-a(x^2+xy+y^2)\right]$$

a. Determine the equilibrium of the system for $a=0$
b. Draw the phase portrait for three different values of a=0, a=1, and a=-1
c. If the scalar function $V(x,y)$ is defined as $V(x,y)=xy(1-x-y)$ show that the time derivatives of V along the system trajectories is given by $\dot{V}=-3axy(1-x-y)\left[(x-1/3)^2+(x-1/3)(y-1/3)+(y-1/3)^2\right]$

Problem 3: For the following systems, find the equilibrium point(s) and determine stability, indicate whether the stability is asymptotic, and whether it is global. Which one can be analyzed through the approximated linearized system?

$$\dot{x}=-x^3+\sin^4 x$$
$$\dot{x}=(5-x)^5$$
$$\ddot{x}+\dot{x}^5+x^7=x^2\sin^8 x\cos^2 3x$$
$$\ddot{x}+(x-1)^4\dot{x}^7+x^5=x^3\sin^3 x$$
$$\ddot{x}+(x-1)^2\dot{x}^7+x=\sin(\pi x/2)$$

Problem 4: Consider the nonlinear system

$$\dot{x}=y+x(x^2+y^2-1)\sin\left(\frac{1}{x^2+y^2-1}\right) \qquad \dot{y}=-x+y(x^2+y^2-1)\sin\left(\frac{1}{x^2+y^2-1}\right)$$

Show that the system has infinite number of limit cycles. Determine the stability of these limit cycles.

Problem 5: Equations of motion of a nonlinear second order system is given by

$$\ddot{x}+\alpha(1+\beta x^2)\dot{x}+x(1-\gamma x^2)=0$$

It is intended to study the stability of the system equilibrium points for different values of the real parameters α,β,γ. For different values of these parameters

a) Determine the equilibrium points.
b) Perform the stability analysis for every equilibrium points and specify type of the equilibrium points.
c) Draw appropriate conclusions about stability of the nonlinear system from the approximate linearized system.