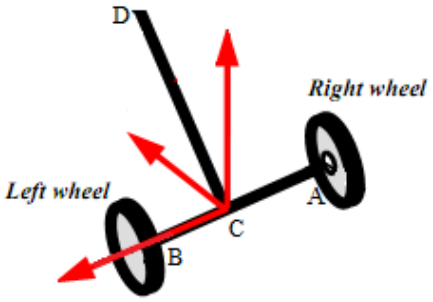
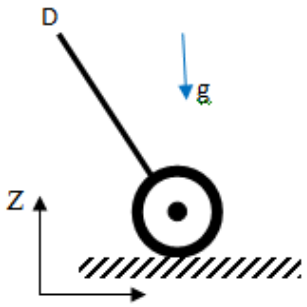
1. Consider the following three systems. For each system:
   - Introduce the generalized coordinates.
   - Drive constraint equations.
   - Calculate number of DOF
   - Specify type of the constraints (holonomic or nonholonomic, scleronomic or rheonomic, and catastatic or acatastatic)
   - Specify type of the system (holonomic or non-holonomic)

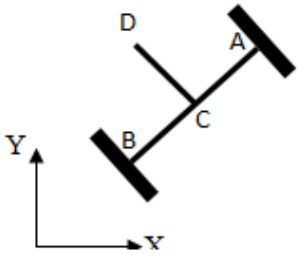
a) **Two Wheeled Inverted Pendulum:** The system consists of a frame and two wheels (right and left wheels) and moves in a horizontal plane. Each wheel rotates with respect to the frame independently.



Two Wheeled Pendulum

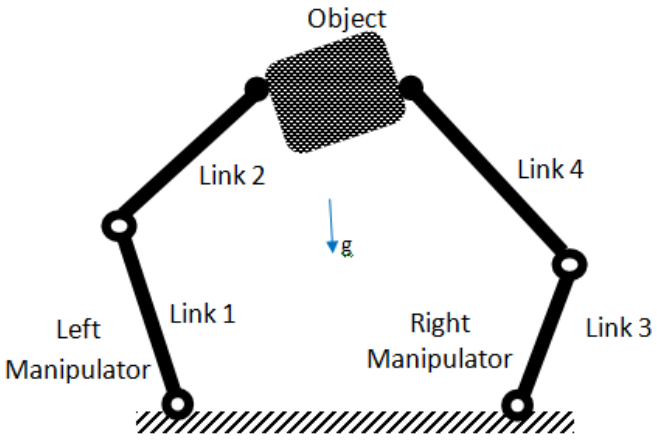


Side View



Top View

b) **Cooperating Manipulators:** Two two-link manipulators move an object through their end-effectors' contact with the object. The contact conditions are assumed to be rough enough that can be considered slip-free contacts. The whole system moves in a vertical plane.



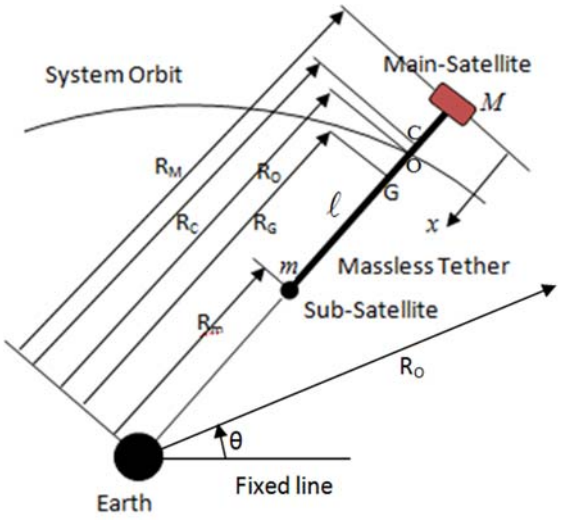
Cooperating Manipulators

c) **Space Elevator System:** The system consists of a main-satellite which deploys a sub-satellite by a massless tether. The whole system is moving on a known orbit (circular or elliptic orbit) around the Earth (with constant angular velocity). The tether is assumed to be rigid and has no librational motion. There are three significant points on the tether line. Center of mass (C), center of gravity (G), and center of orbit (O). R_O is the system center of orbit and has the mentioned orbital motion. The magnitude of gravitational forces for the

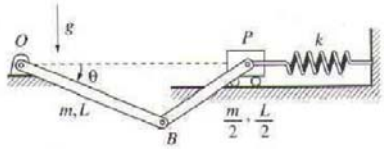
main-satellite and sub-satellite is given by $W_M = -\mu M / R_M^3$ and $W_m = -\mu m / R_m^3$. Directions of these forces are toward the Earth. By definition

$$x_C = m\ell/(M+m)\,, \qquad \frac{M}{r^3} + \frac{m}{(r-\ell)^3} = \frac{M+m}{(r-x_G)^3}$$

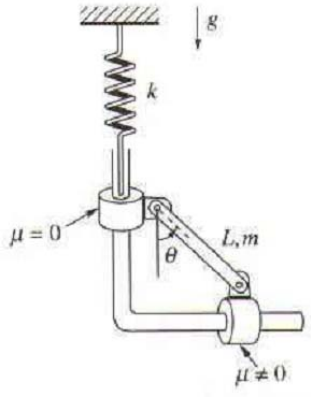
$$R_M = r\,, \quad R_m = r-\ell\,, \quad R_C = r-x_c\,, \quad R_G = r-x_G\,, \quad R_O = \sqrt[3]{R_C R_G^2}$$



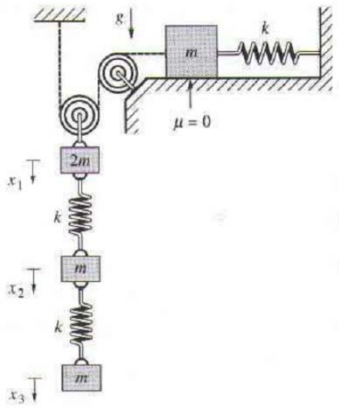
2. Consider the three systems shown below. For each system:
   - Determine the equilibrium point using virtual work principle.
   - Derive the equations of motion using Hamilton's principle.



a- Slider crank system



b- Sliding rod system



c- Pully system