

# Optimal Trajectory Planning for a Cooperative Manipulator with Flexible Joints

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**Abstract**— The optimal path planning for two cooperating manipulators with flexible joints carrying a solid object on a prescribed tip trajectory has been studied using kinematic resolution. The formulation has been derived using Pontryagin Minimum Principle which results in a Two Point Boundary Value Problem (TPBVP). The problem has been solved for globally minimized elastic deformation of flexible joints while the end effector moves along the specified path. The problem has been also solved for minimization of joints velocity along the entire path and the results are compared.

## I. INTRODUCTION

MOST of industrial robots in use today, are composed of heavy and stiff links to satisfy the required accuracy in robot motion. These links, therefore, have inherently large inertia and require in turn more time and power to complete the motion. Hence most existing manipulators have very low payload to total weight ratio. To increase productivity by fast motion and to have less energy consumption, robot arms are required to have light and consequently flexible structures. Flexibility in joints is another important problem which exists in almost all of the robotic systems. The vibration produced by these flexible joints and arms causes difficulty in the robot motion and leads to error in motion of the manipulator tip especially when it reaches to the desired end point. So an effective method should be employed to reduce this undesired vibration.

Cooperation among robots to perform a common task has created a new field of study in Robotics. The capability of cooperative robot systems to perform complicated, accurate and high performance functions, not expected of single robots such as high payload to total weight ratio, has attracted a lot of attention. When two robots are cooperating, they create a closed kinematic chain. Regardless of the kind of object grasping, the closed chain is usually redundant.

Although there has been much work in the area of controlling flexible manipulators to follow a desired trajectory [1-4], a little work has been done in planning the trajectory itself while the flexibility is considered. Such a

planning is necessary either for a point to point motion or for resolving the redundancy of redundant robots to move along a given trajectory. However, as a result of kinematic redundancy, path planning for redundant manipulators is a complicated job. The common idea in redundancy resolution which deals with selecting a single configuration among all possible ones for redundant manipulators is that redundancy should be resolved in such a way that the mechanism optimizes a performance index of system while carrying out its given task. Use of redundancy in rigid robot manipulators has been extensively examined. Keshmiri and Hosseini developed a scheme for the optimal path planning for rigid redundant cooperative manipulators carrying an object on a desired trajectory [5]. However there has been a little work concerning flexible redundant manipulators. Shigang analyzed the effects of initial configuration in vibration reduction of flexible robots with kinematic redundancy [6].

Two possible approaches to resolve the kinematic redundancy are local optimization and global optimization methods. local optimization involves less complexity and requires less computational effort and it is more suitable for real time implementation. Global optimization, in the sense of minimizing a cost functional along the entire trajectory provides the most desirable measure of the manipulator's performance [7-11]. Accordingly, to obtain the optimal solution, one should solve a boundary value problem. Other authors attempted to find the optimal path by solving parameter optimization problem extracted from abstract optimization problem. Keshmiri and Hosseini used a direct method for path planning of rigid cooperative robot systems [12]. In the previous works, some schemes have been developed on optimal path generation for flexible redundant manipulators by local optimization of some objective functions [13]. In this paper a method of motion planning is proposed to obtain an optimal self motion for two cooperating robot manipulators with flexible joints by a two point boundary value problem (TPBV) where globally minimizes the elastic deformations along a specified path. The kinematic index of deflection in flexible joints is used in this study and kinematic redundancy of the closed chain of cooperative robots is solved based on Optimal Control Theory.

Next section of the paper is dedicated to explain the equations of motion for a general cooperative robot system with flexible joints. The formulation of optimal trajectory planning is developed in section 3. Section 4 describes illustrative examples in order to demonstrate the effectiveness of the proposed approach. Finally, section 5

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