

ON THE PLANAR MOTION IN THE FULL TWO-BODY PROBLEM

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The motion of binary asteroids, modeled as the full two-body problem, is studied, considering shape and mass distribution of the bodies. Using the Lagrangian approach, the equations governing the planar motion are derived. The resulting system of four equations is nonlinear and coupled. These equations are solved numerically. In the particular case where the bodies are axisymmetric around an axis normal to the plane, the system reduces to a single equation, with small nonlinearity. The method of multiple scales is used to obtain a first-order solution for the reduced nonlinear equation. This is shown to be sufficient when compared with the numerical solution. Example cases include peanut-shaped bodies.

INTRODUCTION

Since the detection of a natural satellite around the asteroid Ida (during a spacecraft flyby in 1993), numerous asteroid pairs have been observed in the Solar System. According to Margot et al.¹, such binary asteroids are common, making up around 16% of near-Earth asteroids and about 2% of the main asteroid belt. With continuing interest in asteroid exploration for scientific purposes, it is likely that some future missions will be to binary asteroid systems. Therefore, it is of interest to study the motion of a satellite in the vicinity of such systems. Before that, the interaction between the two bodies of an asteroid pair must be investigated; this is the objective of the current paper.

Early work specifically on the dynamics of binary asteroids includes a two-part paper by Chauvineau and Mignard^{2,3}, who modeled the system as Hill's three-body problem. The problem consists of studying the motion of two small particles (the binary asteroid) under the influence of their mutual attraction and of a massive distant body (the Sun).

When the two bodies are close enough, their mutual attraction dominates their motion and the effect of the distant body's gravitation can be neglected. In this case, the system can be reduced to a two-body problem. In the classical literature, the system under consideration is composed of point-masses; such systems have been studied extensively over the years. However, asteroids are usually non-spherical; their shape, mass distribution, and orientation must be considered. This is usually called the "full" two-body problem. In a paper by Maciejewski⁴, a formulation for the Newton-Euler equations for three-dimensional motion was derived, in vector-matrix form. These

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