

# GUAS and UUB Robust Robot Controllers with Joint Position and Velocity Dependent Unstructured Uncertainty Bound

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**Abstract** - In this paper, we present robust control laws for robot manipulators to track a desired trajectory in the presence of both parametric and unstructured uncertainties. The bound on parametric uncertainty is considered as a constant and the bound on unstructured uncertainty is supposed to include a linear combination of norms the joint position and velocity vectors. These robust control laws include a PD feedback part, a dynamic feedforward compensator with nominal robot parameters and two robust dynamic feedforward compensations for parametric and unstructured uncertainties. The stability of the closed-loop system is established by the Lyapunov function. The system with the first proposed control law is prove to be global uniform asymptotic stable (GUAS) and the system with the second control law is prove to have uniformly ultimately bounded (UUB) tracking error. The radius of the ultimated boundedness set is also calculated. The tracking performance of this approach is satisfactory as the simulations carried out on a planar manipulator show.

**Index Terms** - Robust Control, Robot Manipulator, Lyapunov Direct Method, Uniform Stability, Ultimately Boundedness

## I. INTRODUCTION

Uncertainty denotes any obscure element in the dynamics of the real system [1]. In real environment manipulators are usually subjected to uncertainties such as variable payloads, friction torques, unmodeled dynamics, torque disturbances, etc that degrade the system responses.

There are basically two underlying philosophies to the control of uncertain systems: the adaptive control philosophy, and the robust control philosophy. In the adaptive approach, one designs a controller which attempts to *learn* the uncertain parameters of particular system and, if properly designed will eventually be a *best* controller for the system in question. In the robust approach, the controller has a fixed-structure which yields *acceptable* performance for a given plant uncertainty set. In general, the adaptive approach is applicable to a wider range of uncertainties, but in robust controllers no time is required to *tune* the controller to the plant variations [2].

Robust control does not need the exact functional natures of model [3] and guarantees stability and performance of uncertain systems. Its design only requires some knowledge

about bounding functions on the largest possible size of the uncertainty [1]. Spong introduced an approach [4] that makes the system robust to parametric uncertainty. He considered a constant bound for parametric uncertainty. In [5], we introduced a robust approach that can compensate for not only parametric uncertainty but also unstructured uncertainty.

In this paper, we present an approach for compensating for both parametric and unstructured uncertainties by considering an upper bound for unstructured uncertainty that include a linear combination of joint position and velocity norms. We compare the response of the Spong's approach and our approach by simulation and present the results. The results of the proposed approach can be used to improve the robustness of our previous works [6]-[8] to unstructured uncertainty.

## II. ROBUST MANIPULATOR CONTROL

### A. Manipulator Model

A rigid link manipulator can be described by

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + \tau_d = \tau \quad (1)$$

where  $q$  is the  $n \times 1$  vector of joint displacements,  $\tau$  is the  $n \times 1$  vector of applied joint torques (or forces),  $M(q)$  is the  $n \times n$  symmetric positive definite (pd) manipulator inertia matrix,  $V_m(q, \dot{q})\dot{q}$  is the  $n \times 1$  vector of centripetal or Coriolis torques, and  $G(q)$  is the  $n \times 1$  vector of gravitational torques.

Two simplifying properties should be noted about the above dynamic structure. First, the two  $n \times n$  matrices  $M$  and  $V_m$  are not independent. Specially, given a proper definition of the matrix  $V_m$  (note that the centripetal and Coriolis torque vector  $V_m\dot{q}$  is uniquely defined, but that the matrix  $V_m$  is not), the matrix  $\dot{M} - 2V_m$  is skew-symmetric, a property which can be easily derived from the Lagrangian formulation of the manipulator dynamics and which reflects conservation of energy. This property can also be written  $\dot{M} = V_m + V_m^T$  since  $\dot{M}$  is symmetric. The second important property is that the individual terms on the left-