A Note on a Reduced-Order Observer Based Controller for a Class of Lipschitz Nonlinear Systems

Mohamadreza Homayounzade[1]
Ph.D. Student
e-mail: m.homayounzade@me.iut.ac.ir

Mehdi Keshmiri
Associate Professor
e-mail: mehdik@cc.iut.ac.ir

Department of Mechanical Engineering,
Isfahan University of Technology,
P.O. Box 84156,
Isfahan, Iran

This paper presents a novel reduced-order observer based controller for a class of Lipschitz nonlinear systems, described by a set of second order ordinary differential equations. The control law is designed based on the measured output and estimated states. The main features are: (1) The computation cost is reduced noticeably, since the observer is a reduced-order one; (2) The controller guarantees semi-global exponential stability for both estimation and tracking error; and (3) The proposed method can be used in a large range of applications, especially in mechanical systems. The effectiveness of the proposed method is investigated through the numerical simulation for a two-degrees-of-freedom robot manipulator acting on a horizontal worktable. [DOI: 10.1115/1.4007235]

1 Introduction

Observer design for nonlinear systems has been a very active field of research due to its importance in many practical applications (for example, see Misawa and Hedrick [1]).

Observer based feedback controller design generally involves two interrelated stages; observer design and controller design. In order to cope with the problem of observer and controller design for nonlinear systems, it is usual to consider special classes of nonlinear systems. Two particular classes of systems frequently referred to in the literature are Lipschitz nonlinear systems [2], and nonlinear systems with a triangular structure [3].

Roughly speaking, there are four approaches to nonlinear observer design. The first one is based on transforming the nonlinear system into a so called observer normal form causing the dynamic error of states to be linear so that the state observer can be designed utilizing linear techniques [4]. In the second approach, the system dynamics is sub divided into a linear part, which is presumed to be observable from the system output and a nonlinear part which is supposedly locally or globally Lipschitz [5,6]. Thus observer design can be easily performed through solving a Lyapunov or Riccati equation [7].

The third and fourth approaches are the variable structure observer [8], and high gain observer methods [9]. Similar to linear systems, the state observers of nonlinear systems can be classified into full-order and reduced-order observers. A reduced-order observer only estimates part of states that are independent of the system output, and consequently, it has a lower dimension than that of a full-order observer [10,11].

Unlike linear systems, separation principle in observer and controller does not generally hold for nonlinear systems indicating that the observer and controller cannot be designed individually. Therefore, feedback control problems for nonlinear systems with output unmeasured states are much more complicated than a case of full state measurement.

Observer based controller design have been widely studied. The readers are referred to the available literature, see, for example Ref. [8], [12–16], however the observers are full-order ones.

In this paper, for a class of Lipschitz nonlinear systems, a new reduced-order observer based controller is introduced. Compared to the previous ones, in addition to reducing the observer order, and removing some of the limitations in the system dynamics and control law, the proposed observer based controller benefits from the system global exponential stability, both in estimation performance and control performance.

The rest of the paper is organized as follows. In Sec. 2, mathematical notations are presented. In Sec. 3 the reduced-order observer is designed initially. In Sec. 4 the observer based controller is designed and the system stability, both in the state estimation and the tracking error is analyzed. Section 5 is devoted to the implementation of the proposed controller to a constrained robotic system and for the simulation purposes a two link constrained planar robot is considered. The concluding comments are included in Sec. 6.

2 Preliminaries

In this section, a class of nonlinear systems is considered and their governing equations are presented. Let us consider a class of dynamic systems described by a set of second order ordinary differential equation (ODE)

$$\ddot{\boldsymbol{\varepsilon}} = \mathbf{f}(\boldsymbol{\varepsilon}, \dot{\boldsymbol{\varepsilon}}, t) + \mathbf{g}(\mathbf{y}, t)\mathbf{u} \tag{1}$$

where $\boldsymbol{\varepsilon}$ & $\dot{\boldsymbol{\varepsilon}} \in \Re^n$ represent the system states, $\mathbf{y} \in \Re^m$ is the system output assumed to be a linear combination of the states; $\mathbf{u} \in \Re^n$ is the input control vector and $\mathbf{g} \in \Re^{n\times n}$ is assumed to be an invertible matrix. The nonlinear term $\mathbf{f}(\mathbf{x}, t)$ is assumed to be a Lipschitz functional vector with respect to state vector $\mathbf{x} = [\boldsymbol{\varepsilon}^T, \dot{\boldsymbol{\varepsilon}}^T]^T$.

Remark: Let an arbitrary function $\mathbf{f}(t, \mathbf{x}) : [a, b] \times D \to \Re^m$ be locally Lipschitz on $D \subset \Re^n$. Suppose that $[\partial\mathbf{f}/\partial\mathbf{x}]$ exists and is continuous on $[a, b] \times D$. If for a convex subset $W \subset D$, there is a constant $\gamma \geq 0$ such that $\|(\partial\mathbf{f}(t, \mathbf{x}))/\partial\mathbf{x}\| \leq \gamma$ on $[a, b] \times D$, then

$$\|\mathbf{f}(t, \mathbf{x}) - \mathbf{f}(t, \mathbf{y})\| \leq \gamma\|\mathbf{x} - \mathbf{y}\| \tag{2}$$

for all $t \in [a, b]$, $\mathbf{x} \in W$ and $\mathbf{y} \in W$ [2].

Denoting the system tracking error by $\mathbf{e} = \mathbf{x} - \mathbf{x}_d$, where $\mathbf{x}_d = [\boldsymbol{\varepsilon}_d^T, \dot{\boldsymbol{\varepsilon}}_d^T]^T$ is the desired state vector, the governing equations for the error dynamics can be written as

$$\begin{aligned} \dot{\mathbf{e}} &= \mathbf{A_c}\mathbf{e} + \mathbf{B_c}(\mathbf{f}(\mathbf{x}, t) + \mathbf{g}(\mathbf{y}, t)\mathbf{u} - \ddot{\boldsymbol{\varepsilon}}_d) \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned} \tag{3}$$

where

$$\mathbf{A_c} = \begin{bmatrix} 0 & \mathbf{I} \\ 0 & 0 \end{bmatrix}, \quad \mathbf{B_c} = \begin{bmatrix} 0 \\ \mathbf{I} \end{bmatrix} \tag{4}$$

The pair $(\mathbf{A_c}, \mathbf{C})$ is assumed to be observable and the matrix $\mathbf{C}$ to be a full row rank matrix.

Definition: *For an arbitrary positive definite (P.D.) or negative definite (N.D.) symmetric matrix such as* $\mathbf{Y}$*, throughout this paper by* Y_m *and* Y_M*, the authors mean the minimum and maximum eigenvalues of that matrix. Hence we can state*

$$Y_m\|\mathbf{x}\|^2 \leq \mathbf{x}^{\mathbf{T}}\mathbf{Y}\mathbf{x} \leq Y_M\|\mathbf{x}\|^2 \tag{5}$$

3 Observer Design

In order to estimate the states from the system output $\mathbf{y}$, an observer is designed in the following. Consider an observer defined by

[1]Corresponding author.

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